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Spatial evolution of cylindrical beams in complex media

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Abstract
The spatial evolution of vector electromagnetic fields in bianisotropic cylindrically symmetric structures is studied. We introduce evolution operators (characteristic matrices), impedance tensors, reflection and transmission operators of cylindrical beams which are convenient for describing waves in layered media. We predict the existence of fractional Bessel beams with integer topological charge and cylindrically symmetric beams which can be presented as the product of the Bessel function and the exponent. We investigate energy and polarization characteristics of such beams.

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1. Introduction

The study of electromagnetic fields with cylindrical symmetry has been a distinct area of optics for nearly 20 years. An example of a cylindrical beam is a Bessel beam [1–4]. Nondiffracting Bessel beams possess specific properties, such as the ring-shaped angular spectrum and the beam reconstruction. Nowadays, a topical subject of research is the investigation of higher-order fractional Bessel beams generated from a spatial light modulator [5–8]. The existence of such beams is connected with azimuthal phase variation at non-integer multiples of 2π. A fractional Bessel beam has an intensity distribution with opening slit in the ring pattern. By changing the Bessel beam hologram from the fractional order to the integer one, the opening slit can be closed. Such opening and closing of the slit is expected to allow the separation of microparticles. Another type of cylindrical beams are the Bessel–Gauss beams which exhibit the properties of both nondiffracting and Gauss beams. Usually, the scalar Bessel–Gauss fields in the paraxial approximation are studied, although there are papers in which vector [9, 10] and nonparaxial [11] beams are analysed. Vector beams can be generated in few-mode fibres excited by Laguerre–Gauss beams [12]. The Bessel–Gauss beams are applied in microscopy, lithography, material processing, microellipsometry and spectroscopy.

In this work we consider cylindrically symmetric beams, which appear as solutions of the Maxwell equations in bianisotropic media. Bessel and Bessel–Gauss beams in anisotropic media have been already investigated. In the papers [13, 14] the transformation of the Bessel beam order in uniaxial and biaxial crystals was predicted. The authors of [14] showed that almost the whole energy of the input Bessel beam of the zero order can be converted into the Bessel beam of the second order. In [15, 16], linearly polarized cylindrically symmetric fields (Bessel, Bessel–Gauss and Laguerre–Gauss beams) in uniaxial crystals were studied.

This paper is organized as follows. Section 2 is devoted to a brief summary of the operator method for cylindrical electromagnetic waves which was developed in [17–19] and applied there for describing the modes in optical fibres. Using the operator solutions, in section 3 vector cylindrical beams in bianisitropic media are formed. In section 4 the impedance tensors, spatial evolution operators (classical propagators, characteristic matrices), reflection and transmission operators for cylindrical beams in multi-layered media are determined. Examples of cylindrically symmetric beams are considered in section 5. In particular, we investigate the energy and polarization characteristics of the integer and fractional Bessel beams, waves with radial dependence in the form of the product of the Bessel function and the exponent. In section 6, we present the main results obtained in the paper and possible directions for further investigations.

2. Cylindrical waves in bianisotropic media

We consider the electromagnetic waves satisfying the classical Maxwell equations in bianisotropic media with constitutive equations

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} + \alpha \boldsymbol{H} \qquad \boldsymbol{B} = \kappa \boldsymbol{E} + \mu \boldsymbol{H} \tag{1}$$

where $\boldsymbol{H}$, $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{D}$ are the strengths and inductions of the electric and magnetic fields. The dielectric permittivity tensor ε, magnetic permeability tensor μ and gyration pseudotensors α, κ can be decomposed with dyads made up on basis of sorts of cylindrical coordinates (r, φ, z) with constant coefficients:

$$\xi = \sum_{i,j=1}^{3} \xi_{ij}\, \boldsymbol{e}_i(\varphi) \otimes \boldsymbol{e}_j(\varphi) \tag{2}$$

where ξ is one of the tensors $\varepsilon, \mu, \alpha, \kappa$; $\boldsymbol{e}_1 = \boldsymbol{e}_r(\varphi)$, $\boldsymbol{e}_2 = \boldsymbol{e}_\varphi(\varphi)$, $\boldsymbol{e}_3 = \boldsymbol{e}_z$ are the basis vectors of cylindrical coordinates; $\boldsymbol{e}_i \otimes \boldsymbol{e}_j$ is the elementary dyad. Such symmetry of the media under investigation allows us to divide the variables into electric and magnetic field strengths as follows,

$$\begin{pmatrix} \boldsymbol{H}(\boldsymbol{r}, t) \\ \boldsymbol{E}(\boldsymbol{r}, t) \end{pmatrix} = \exp(\mathrm{i}\beta z + \mathrm{i}\nu\varphi - \mathrm{i}\omega t) \begin{pmatrix} \boldsymbol{H}(r, \varphi) \\ \boldsymbol{E}(r, \varphi) \end{pmatrix} \tag{3}$$

where β is the longitudinal wavenumber, ω is the wave frequency, ν is an integer number. Hence, Maxwell's equations can be presented as the system of the ordinary differential equations of the first order [18]:

$$\frac{\mathrm{d}\boldsymbol{W}(r)}{\mathrm{d}r} = \mathrm{i}kM(r)\boldsymbol{W}(r) \tag{4}$$

where

$$
\begin{aligned}
& M=\begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad \boldsymbol{W}=\begin{pmatrix} \boldsymbol{H}_{\mathrm{t}} \\ \boldsymbol{E}_{\mathrm{t}} \end{pmatrix} \\
& A=\frac{\mathrm{i}}{kr}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi+\boldsymbol{e}_r^{\times}\alpha I+\boldsymbol{e}_r^{\times}\varepsilon \boldsymbol{e}_r \otimes \boldsymbol{v}_3+\boldsymbol{e}_r^{\times}(\boldsymbol{u}+\alpha \boldsymbol{e}_r)\otimes \boldsymbol{v}_1 \\
& B=\boldsymbol{e}_r^{\times}\varepsilon I+\boldsymbol{e}_r^{\times}\varepsilon \boldsymbol{e}_r \otimes \boldsymbol{v}_4+\boldsymbol{e}_r^{\times}(\boldsymbol{u}+\alpha \boldsymbol{e}_r)\otimes \boldsymbol{v}_2 \\
& C=-\boldsymbol{e}_r^{\times}\mu I-\boldsymbol{e}_r^{\times}\mu \boldsymbol{e}_r \otimes \boldsymbol{v}_1+\boldsymbol{e}_r^{\times}(\boldsymbol{u}-\kappa \boldsymbol{e}_r)\otimes \boldsymbol{v}_3 \\
& D=\frac{\mathrm{i}}{kr}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi-\boldsymbol{e}_r^{\times}\kappa I-\boldsymbol{e}_r^{\times}\mu \boldsymbol{e}_r \otimes \boldsymbol{v}_2+\boldsymbol{e}_r^{\times}(\boldsymbol{u}-\kappa \boldsymbol{e}_r)\otimes \boldsymbol{v}_4 .
\end{aligned} \tag{5}
$$

$\boldsymbol{E}_{\mathrm{t}} = I_r\boldsymbol{E}$ and $\boldsymbol{H}_{\mathrm{t}} = I_r\boldsymbol{H}$ are the tangential field components, $I_r = 1 - \boldsymbol{e}_r \otimes \boldsymbol{e}_r$ is the projection operator onto the plane orthogonal to the unit vector $\boldsymbol{e}_r$, $\boldsymbol{e}_r^{\times}$ is the tensor dual to the vector $\boldsymbol{e}_r$ [20, 21], $k = \omega/c$. Tangential components include two projections, one of which is longitudinal projection (E_z, H_z) and another is the azimuthal one (E_φ, H_φ). Tangential components enable us to restore the total fields as

$$
\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{E} \end{pmatrix}=V\begin{pmatrix} \boldsymbol{H}_{\mathrm{t}} \\ \boldsymbol{E}_{\mathrm{t}} \end{pmatrix} \qquad V=\begin{pmatrix} I+\boldsymbol{e}_r \otimes \boldsymbol{v}_1 & \boldsymbol{e}_r \otimes \boldsymbol{v}_2 \\ \boldsymbol{e}_r \otimes \boldsymbol{v}_3 & I+\boldsymbol{e}_r \otimes \boldsymbol{v}_4 \end{pmatrix} \tag{6}
$$

where the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4, \boldsymbol{u}$ are equal to

$$
\begin{aligned}
& \boldsymbol{v}_1=\delta_r(\kappa_{rr}\boldsymbol{e}_r\alpha I_r-\varepsilon_{rr}\boldsymbol{e}_r\mu I_r-\kappa_{rr}\boldsymbol{u}) \qquad \boldsymbol{v}_2=\delta_r(\kappa_{rr}\boldsymbol{e}_r\varepsilon I_r-\varepsilon_{rr}\boldsymbol{e}_r\kappa I_r-\varepsilon_{rr}\boldsymbol{u}) \\
& \boldsymbol{v}_3=\delta_r(\alpha_{rr}\boldsymbol{e}_r\mu I_r-\mu_{rr}\boldsymbol{e}_r\alpha I_r+\mu_{rr}\boldsymbol{u}) \qquad \boldsymbol{v}_4=\delta_r(\alpha_{rr}\boldsymbol{e}_r\kappa I_r-\mu_{rr}\boldsymbol{e}_r\varepsilon I_r+\alpha_{rr}\boldsymbol{u}) \\
& \boldsymbol{u}=(\beta/k)\boldsymbol{e}_\varphi-\nu/(kr)\boldsymbol{e}_z \qquad \delta_r=(\varepsilon_{rr}\mu_{rr}-\alpha_{rr}\kappa_{rr})^{-1} \\
& \varepsilon_{rr}=\boldsymbol{e}_r\varepsilon\boldsymbol{e}_r \qquad \mu_{rr}=\boldsymbol{e}_r\mu\boldsymbol{e}_r \qquad \alpha_{rr}=\boldsymbol{e}_r\alpha\boldsymbol{e}_r \qquad \kappa_{rr}=\boldsymbol{e}_r\kappa\boldsymbol{e}_r .
\end{aligned} \tag{7}
$$

Matrix M can be written as a power expansion of $1/r$:

$$
M=M^{(0)}+\frac{1}{r}M^{(1)}+\frac{1}{r^2}M^{(2)} \tag{8}
$$

where $M^{(0)}, M^{(1)}, M^{(2)}$ are the constant matrices. Then we will present the tangential field components $\boldsymbol{W}$ and block matrix M as decomposition of the basis vectors of cylindrical coordinates

$$
\boldsymbol{W}=\vec{w}_\varphi\boldsymbol{e}_\varphi+\vec{w}_z\boldsymbol{e}_z \qquad \vec{w}_\varphi=\boldsymbol{e}_\varphi\boldsymbol{W}=\begin{pmatrix} \boldsymbol{e}_\varphi\boldsymbol{H} \\ \boldsymbol{e}_\varphi\boldsymbol{E} \end{pmatrix}=\begin{pmatrix} H_\varphi \\ E_\varphi \end{pmatrix} \qquad \vec{w}_z=\begin{pmatrix} H_z \\ E_z \end{pmatrix}
$$

$$
\begin{aligned}
& M=M_{zz}\boldsymbol{e}_z\otimes\boldsymbol{e}_z+M_{z\varphi}\boldsymbol{e}_z\otimes\boldsymbol{e}_\varphi+M_{\varphi z}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_z+M_{\varphi\varphi}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi \\
& M_{zz}=\boldsymbol{e}_zM\boldsymbol{e}_z=\begin{pmatrix} \boldsymbol{e}_zA\boldsymbol{e}_z & \boldsymbol{e}_zB\boldsymbol{e}_z \\ \boldsymbol{e}_zC\boldsymbol{e}_z & \boldsymbol{e}_zD\boldsymbol{e}_z \end{pmatrix}=\begin{pmatrix} A_{zz} & B_{zz} \\ C_{zz} & D_{zz} \end{pmatrix} \\
& M_{z\varphi}=\boldsymbol{e}_zM\boldsymbol{e}_\varphi \qquad M_{\varphi z}=\boldsymbol{e}_\varphi M\boldsymbol{e}_z \qquad M_{\varphi\varphi}=\boldsymbol{e}_\varphi M\boldsymbol{e}_\varphi .
\end{aligned}
$$

Here $M_{zz}, M_{z\varphi}, M_{\varphi z}$ and $M_{\varphi\varphi}$ are the matrices of the two-dimensional space. For bianisotropic media under consideration the constant matrices $M^{(0)}, M^{(1)}, M^{(2)}$ are equal to

$$
\begin{aligned}
& M^{(0)}=M_{zz}^{(0)}\boldsymbol{e}_z\otimes\boldsymbol{e}_z+M_{z\varphi}^{(0)}\boldsymbol{e}_z\otimes\boldsymbol{e}_\varphi+M_{\varphi z}^{(0)}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_z+M_{\varphi\varphi}^{(0)}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi \\
& M^{(1)}=M_{zz}^{(1)}\boldsymbol{e}_z\otimes\boldsymbol{e}_z+M_{\varphi z}^{(1)}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_z+M_{\varphi\varphi}^{(1)}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi \qquad M^{(2)}=M_{\varphi z}^{(2)}\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_z .
\end{aligned} \tag{9}
$$

From equation (4) it follows the differential equation of the second order

$$
\vec{w}_z''+\left(P^{(0)}+\frac{1}{r}P^{(1)}\right)\vec{w}_z'+\left(Q^{(0)}+\frac{1}{r}Q^{(1)}+\frac{1}{r^2}Q^{(2)}\right)\vec{w}_z=0 \tag{10}
$$

where the prime denotes the r-derivative. Constant 2×2 matrices $P^{(0)}$, $P^{(1)}$, $Q^{(0)}$, $Q^{(1)}$, $Q^{(2)}$ equal

$$\begin{aligned}
P^{(0)} &= -\mathrm{i}k\big(M_{zz}^{(0)} + M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(0)} M_{z\varphi}^{(0)-1}\big)\\
P^{(1)} &= -\mathrm{i}k\big(M_{zz}^{(1)} + M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(1)} M_{z\varphi}^{(0)-1}\big)\\
Q^{(0)} &= k^2\big(M_{z\varphi}^{(0)} M_{\varphi z}^{(0)} - M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(0)} M_{z\varphi}^{(0)-1} M_{zz}^{(0)}\big)\\
Q^{(1)} &= k^2\big(M_{z\varphi}^{(0)} M_{\varphi z}^{(1)} - M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(1)} M_{z\varphi}^{(0)-1} M_{zz}^{(0)} - M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(0)} M_{z\varphi}^{(0)-1} M_{zz}^{(1)}\big)\\
Q^{(2)} &= \mathrm{i}k M_{zz}^{(1)} + k^2 M_{z\varphi}^{(0)} M_{\varphi z}^{(2)} - k^2 M_{z\varphi}^{(0)} M_{\varphi\varphi}^{(1)} M_{z\varphi}^{(0)-1} M_{zz}^{(1)}.
\end{aligned} \tag{11}$$

Generally, the solution of equation (10) is the linear combination of four independent solutions:

$$\vec{w}_z(r) = \sum_{j=1}^{4} T_j(r) c_j \vec{a}_j \tag{12}$$

where $T_j(r)$ are the solutions expressed by the 2×2 matrices, $\vec{a}_j$ are arbitrary vectors, c_j are constants. On the basis of the constants c_j the vectors of the three-dimensional space $\boldsymbol{c}_1 = c_1\boldsymbol{e}_z + c_2\boldsymbol{e}_\varphi$ and $\boldsymbol{c}_2 = c_3\boldsymbol{e}_z + c_4\boldsymbol{e}_\varphi$ are introduced, then the tangential field components $\boldsymbol{W}$ take the form

$$\boldsymbol{W} = \begin{pmatrix} \eta_1 \boldsymbol{c}_1 \\ \zeta_1 \boldsymbol{c}_1 \end{pmatrix} + \begin{pmatrix} \eta_2 \boldsymbol{c}_2 \\ \zeta_2 \boldsymbol{c}_2 \end{pmatrix} \tag{13}$$

where the tensors η_1, η_2, ζ_1, ζ_2 equal

$$\begin{aligned}
\eta_1 &= \vec{e}_1 T_1 \vec{a}_1 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_1 \hat{Z} T_1 \vec{a}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z + \vec{e}_1 T_2 \vec{a}_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_1 \hat{Z} T_2 \vec{a}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi\\
\eta_2 &= \vec{e}_1 T_3 \vec{a}_3 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_1 \hat{Z} T_3 \vec{a}_3 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z + \vec{e}_1 T_4 \vec{a}_4 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_1 \hat{Z} T_4 \vec{a}_4 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi\\
\zeta_1 &= \vec{e}_2 T_1 \vec{a}_1 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_2 \hat{Z} T_1 \vec{a}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z + \vec{e}_2 T_2 \vec{a}_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_2 \hat{Z} T_2 \vec{a}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi\\
\zeta_2 &= \vec{e}_2 T_3 \vec{a}_3 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_2 \hat{Z} T_3 \vec{a}_3 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z + \vec{e}_2 T_4 \vec{a}_4 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_2 \hat{Z} T_4 \vec{a}_4 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi.
\end{aligned} \tag{14}$$

Here we use the differential operator

$$\hat{Z} = M_{z\varphi}^{-1}\left(\frac{1}{\mathrm{i}k}\frac{\mathrm{d}}{\mathrm{d}r} - M_{zz}\right)$$

and unit vectors of the two-dimensional space

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The waves (13) are called cylindrical waves, because their amplitudes depend only on the radial coordinate r. The azimuthal coordinate φ enters only into the basis vectors of cylindrical coordinates which determine the vector structure of electromagnetic field. During the spatial evolution of the waves the basis vectors are considered to be constant. In general, in a bianisotropic medium there are four cylindrical waves which correspond to the independent solutions for the longitudinal field components T_j. The tangential field components are written using the tensors η and ζ, each of which is formed by the couple of solutions T. Such notation is connected with the plane wave analogy. Four plane waves can be also separated: one couple of waves is forward and another is backward propagating.

3. Vector cylindrical beams

Cylindrical beams propagating along the z axis are the exact solutions of the Maxwell equations in cylindrical coordinates (r, φ, z), which are limited at any point of the beam cross section (i.e. both for $r = 0$ and $r = \infty$). Undoubtedly, such beams are vector fields. In this section, we will consider the general principles for theoretically constructing the beams in complex media of the form (2), the particular cases of solutions in which being the Bessel beams.

In section 2, the general solutions were obtained which are expressed by the 2×2-matrices T for longitudinal field components, and tensors η and ζ for tangential components. The tangential components $\boldsymbol{H}_\mathrm{t}$ and $\boldsymbol{E}_\mathrm{t}$ are convenient for describing circular fibre modes, because they are continuous on the cylindrical interface between two media. It is well known that such waves as waveguide modes are nondiffracting fields. This result follows from the structure of solution $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}(x, y)\exp(\mathrm{i}\beta z)$, in which longitudinal and transverse coordinates are divided, and the transversal intensity distribution is an invariant during propagation. When we wish to form beam solutions, we should take into account some important features. First, we consider medium of the form (2) which is infinite in the transverse cross section. That is why the tangential field components $\boldsymbol{H}_\mathrm{t}$ and $\boldsymbol{E}_\mathrm{t}$ are improper for the use and it is necessary to introduce new transverse field components which lie in the plane $z =$ const and are continuous in the plane interfaces. Second, the solutions should be chosen correctly. The longitudinal field components are determined by the four solutions T_j, only two of which are usually applied for describing the beam. The remaining solutions give infinite field strengths in the beam centre or at infinity. For example, in isotropic medium there are some sets of cylindrically symmetric solutions T which are expressed in terms of the Bessel functions of the first and second kinds, or modified Bessel functions, or Hankel functions. But only the Bessel functions of the first kind are solutions for beams propagating in the infinite medium.

Let us suppose that a cylindrical beam is formed by the couple of solutions T_1, T_2. Then in the wave superposition (13) we should assume $\boldsymbol{c}_2 = 0$ to exclude nonphysical solutions. The tangential field components of the beam take the form

$$\begin{pmatrix}\boldsymbol{H}_\mathrm{t}\\ \boldsymbol{E}_\mathrm{t}\end{pmatrix} = \begin{pmatrix}\eta_1\boldsymbol{c}_1\\ \zeta_1\boldsymbol{c}_1\end{pmatrix} \equiv \begin{pmatrix}\eta_1\\ \zeta_1\end{pmatrix}\boldsymbol{c}_1.$$

From the formulae (14) we see that each of the tensors η and ζ can be separated into two parts which are related to the solutions T_1 and T_2, respectively. We will denote them as

$$\eta^{(1)} = \vec{e}_1 T_1 \vec{a}_1 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_1 \hat{Z} T_1 \vec{a}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z \qquad \eta^{(2)} = \vec{e}_1 T_2 \vec{a}_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_1 \hat{Z} T_2 \vec{a}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi$$
$$\zeta^{(1)} = \vec{e}_2 T_1 \vec{a}_1 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_2 \hat{Z} T_1 \vec{a}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z \qquad \zeta^{(2)} = \vec{e}_2 T_2 \vec{a}_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_2 \hat{Z} T_2 \vec{a}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi.$$

Then the tangential field components are written as the following superposition,

$$\begin{pmatrix}\boldsymbol{H}_\mathrm{t}\\ \boldsymbol{E}_\mathrm{t}\end{pmatrix} = \begin{pmatrix}\eta^{(1)} + \eta^{(2)}\\ \zeta^{(1)} + \zeta^{(2)}\end{pmatrix}\boldsymbol{c}_1 = c_1\begin{pmatrix}\eta^{(1)}\boldsymbol{e}_z\\ \zeta^{(1)}\boldsymbol{e}_z\end{pmatrix} + c_2\begin{pmatrix}\eta^{(2)}\boldsymbol{e}_\varphi\\ \zeta^{(2)}\boldsymbol{e}_\varphi\end{pmatrix} \tag{15}$$

where $c_1 = \boldsymbol{c}_1\boldsymbol{e}_z, c_2 = \boldsymbol{c}_1\boldsymbol{e}_\varphi$. Each of the waves (with superscript 1 or 2) is independent: for $c_2 = 0$ the first wave propagates and vice versa. The phase factor $\exp(\mathrm{i}\beta z)$, which is the same for all fibre solutions, takes different values for each independent wave, i.e. the waves are characterized by the longitudinal wavenumbers β_1 and β_2. Therefore, the field strengths of a cylindrically symmetric beam equal

$$\begin{pmatrix}\boldsymbol{H}(\boldsymbol{r})\\ \boldsymbol{E}(\boldsymbol{r})\end{pmatrix} = \exp(\mathrm{i}\nu\varphi + \mathrm{i}\beta_1 z)V(\beta_1)\begin{pmatrix}\eta^{(1)}\boldsymbol{e}_z\\ \zeta^{(1)}\boldsymbol{e}_z\end{pmatrix}c_1 + \exp(\mathrm{i}\nu\varphi + \mathrm{i}\beta_2 z)V(\beta_2)\begin{pmatrix}\eta^{(2)}\boldsymbol{e}_\varphi\\ \zeta^{(2)}\boldsymbol{e}_\varphi\end{pmatrix}c_2 \tag{16}$$

where V is the matrix (6). Further, we introduce the initial field vector in the transverse cross section of the beam $\boldsymbol{a} = c_1\boldsymbol{e}_r + c_2\boldsymbol{e}_\varphi$ and write the electric and magnetic fields as

$$\begin{pmatrix} \boldsymbol{H}(\boldsymbol{r}) \\ \boldsymbol{E}(\boldsymbol{r}) \end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi}\left[\mathrm{e}^{\mathrm{i}\beta_1 z}V(\beta_1)\begin{pmatrix} \eta^{(1)}\boldsymbol{e}_z \otimes \boldsymbol{e}_r \\ \zeta^{(1)}\boldsymbol{e}_z \otimes \boldsymbol{e}_r \end{pmatrix} + \mathrm{e}^{\mathrm{i}\beta_2 z}V(\beta_2)\begin{pmatrix} \eta^{(2)}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi \\ \zeta^{(2)}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi \end{pmatrix}\right]\boldsymbol{a}. \tag{17}$$

We will need transverse field components below: $\boldsymbol{H}_\perp = I_z\boldsymbol{H}$ and $\boldsymbol{e}_z \times \boldsymbol{E}$, where $I_z = 1 - \boldsymbol{e}_z \otimes \boldsymbol{e}_z$ is the projection operator onto the plane normal to the vector $\boldsymbol{e}_z$. Taking into account the form of the matrix V, the transverse components can be expressed by means of planar tensors τ and σ (for a planar tensor τ the equality $\tau I_z = I_z\tau = \tau$ holds true):

$$\begin{pmatrix} \boldsymbol{H}_\perp \\ \boldsymbol{e}_z \times \boldsymbol{E} \end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi}\begin{pmatrix} \tau \\ \sigma \end{pmatrix}\boldsymbol{a} \tag{18}$$

where

$$\begin{aligned} \tau &= \mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{f}_1(r,\varphi) \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{f}_2(r,\varphi) \otimes \boldsymbol{e}_\varphi \\ \sigma &= \mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{g}_1(r,\varphi) \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{g}_2(r,\varphi) \otimes \boldsymbol{e}_\varphi. \end{aligned} \tag{19}$$

Vectors $\boldsymbol{f}$ and $\boldsymbol{g}$ equal

$$\begin{aligned} \boldsymbol{f}_1 &= (\boldsymbol{e}_\varphi\eta^{(1)}\boldsymbol{e}_z)\boldsymbol{e}_\varphi + (\boldsymbol{v}_1(\beta_1)\eta^{(1)}\boldsymbol{e}_z + \boldsymbol{v}_2(\beta_1)\zeta^{(1)}\boldsymbol{e}_z)\boldsymbol{e}_r \\ \boldsymbol{f}_2 &= (\boldsymbol{e}_\varphi\eta^{(2)}\boldsymbol{e}_\varphi)\boldsymbol{e}_\varphi + (\boldsymbol{v}_1(\beta_2)\eta^{(2)}\boldsymbol{e}_\varphi + \boldsymbol{v}_2(\beta_2)\zeta^{(2)}\boldsymbol{e}_\varphi)\boldsymbol{e}_r \\ \boldsymbol{g}_1 &= -(\boldsymbol{e}_\varphi\zeta^{(1)}\boldsymbol{e}_z)\boldsymbol{e}_r + (\boldsymbol{v}_3(\beta_1)\eta^{(1)}\boldsymbol{e}_z + \boldsymbol{v}_4(\beta_1)\zeta^{(1)}\boldsymbol{e}_z)\boldsymbol{e}_\varphi \\ \boldsymbol{g}_2 &= -(\boldsymbol{e}_\varphi\zeta^{(2)}\boldsymbol{e}_\varphi)\boldsymbol{e}_r + (\boldsymbol{v}_3(\beta_2)\eta^{(2)}\boldsymbol{e}_\varphi + \boldsymbol{v}_4(\beta_2)\zeta^{(2)}\boldsymbol{e}_\varphi)\boldsymbol{e}_\varphi. \end{aligned} \tag{20}$$

It is obvious that each independent wave itself is diffraction free. The superposition of the fields with $\beta_1 = \beta_2$ is nondiffracting, too. The beam diverges for the superposition with different longitudinal wavenumbers $\beta_1 \neq \beta_2$.

The time-averaged Poynting vector of the beam in the direction of the unit vector $\boldsymbol{n}$ is calculated according to the formula

$$S_n = \frac{c}{16\pi}(\boldsymbol{H}^* \quad \boldsymbol{E}^*)\begin{pmatrix} 0 & \boldsymbol{n}^\times \\ -\boldsymbol{n}^\times & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{E} \end{pmatrix}. \tag{21}$$

By substituting the cylindrical coordinate basis vectors instead of $\boldsymbol{n}$ one can find radial, azimuthal and longitudinal beam energy flux components. It should be noted that the longitudinal component S_z can be obtained using only transverse electric and magnetic fields.

4. Reflection and transmission operators for cylindrical beams

4.1. Impedance tensors and evolution operators

In a number of problems (for instance, for investigating the wave propagation in layered media) the evolution operators and impedance tensors are found to be useful. Here we will introduce such operators for cylindrical beams under consideration.

The impedance tensor γ satisfies the equation $\boldsymbol{e}_z \times \boldsymbol{E} = \gamma\boldsymbol{H}_\perp$ and equals

$$\gamma = \sigma\tau^- \tag{22}$$

or

$$\gamma = \frac{1}{(\boldsymbol{f}_1 \times \boldsymbol{f}_2)^2}\left[\boldsymbol{g}_1 \otimes \left(-\boldsymbol{f}_2^{\times 2}\boldsymbol{f}_1\right) + \boldsymbol{g}_2 \otimes \left(-\boldsymbol{f}_1^{\times 2}\boldsymbol{f}_2\right)\right]. \tag{23}$$

Further, we will find the link between the amplitudes of electric and magnetic fields during beam propagation in a bianisotropic medium. The transverse field components can be presented as superposition of forward and backward beams:

$$\begin{pmatrix} \boldsymbol{H}_\perp \\ \boldsymbol{e}_z \times \boldsymbol{E} \end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi} \left[\begin{pmatrix} \tau \\ \sigma \end{pmatrix} \boldsymbol{a} + \begin{pmatrix} \tau' \\ \sigma' \end{pmatrix} \boldsymbol{a}' \right]. \tag{24}$$

A backward cylindrical beam propagates in $-z$ direction and is determined by tensors τ', σ', γ' which are obtained from τ, σ, γ by means of the sign variation $\beta_{1,2} \to -\beta_{1,2}$. From the definition of the evolution operator

$$\begin{pmatrix} \boldsymbol{H}_\perp(z) \\ \boldsymbol{e}_z \times \boldsymbol{E}(z) \end{pmatrix} = \Omega_{z_0}^{z} \begin{pmatrix} \boldsymbol{H}_\perp(z_0) \\ \boldsymbol{e}_z \times \boldsymbol{E}(z_0) \end{pmatrix} \tag{25}$$

we can write

$$\Omega_{z_0}^{z} = S(z) S^{-}(z_0) \tag{26}$$

where

$$S(z) = \begin{pmatrix} \tau & \tau' \\ \sigma & \sigma' \end{pmatrix}.$$

The evolution operator and impedance tensor depend on the radial r and angular φ coordinates, although it is not indicated in the notation. As we will see below, the transverse coordinates in these operators determine only polarization vectors and do not influence the beam reflection and transmission coefficients.

4.2. Boundary conditions: beam reflection and transmission operators

Let us consider the plane interface $z = 0$ between two media. The incident and reflected beams propagate in medium 1, and the refracted beam moves in medium 2. The transverse fields are expressed by the formula (18). In notation of the fields of incident, reflected and refracted beams we will use terms without prime, with one prime and with two primes, respectively. Then the continuity conditions for transverse field components at the interface take the form

$$\mathrm{e}^{\mathrm{i}\nu\varphi} \begin{pmatrix} \tau \\ \sigma \end{pmatrix} \boldsymbol{a} + \mathrm{e}^{\mathrm{i}\nu'\varphi} \begin{pmatrix} \tau' \\ \sigma' \end{pmatrix} \boldsymbol{a}' = \mathrm{e}^{\mathrm{i}\nu''\varphi} \begin{pmatrix} \tau'' \\ \sigma'' \end{pmatrix} \boldsymbol{a}''. \tag{27}$$

Since the functions $\mathrm{e}^{\mathrm{i}\nu\varphi}$ are orthogonal, it follows from the boundary conditions that $\nu = \nu' = \nu''$. Magnetic field strengths can be written in terms of the vector functions $\boldsymbol{f}$ (similarly for electric field):

$$\boldsymbol{f}_1 c_1 + \boldsymbol{f}_2 c_2 + \boldsymbol{f}'_1 c'_1 + \boldsymbol{f}'_2 c'_2 = \boldsymbol{f}''_1 c''_1 + \boldsymbol{f}''_2 c''_2 \tag{28}$$

where $\boldsymbol{a} = c_1 \boldsymbol{e}_r + c_2 \boldsymbol{e}_\varphi$. Vectors $\boldsymbol{f}$ depend on the transverse coordinates, mainly on the radial coordinate, since only orts $\boldsymbol{e}_r$ and $\boldsymbol{e}_\varphi$ include the azimuth. Expression (28) should hold true for any coordinate r and constants $c_1, c_2, c'_1, c'_2, c''_1, c''_2$. Therefore, some restrictions on the beams in media 1 and 2 are applied.

The eigenpolarizations of the incident wave $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are noncollinear; they determine the basis in the plane (r, φ). Vectors of the reflected and refracted beams can be decomposed in this basis with some coefficients, which generally depend on the radial coordinate. If the numbers c in (28) are the same for any r, then the decomposition coefficients a and b should be constant to satisfy the boundary conditions (27):

$$\begin{aligned} \boldsymbol{f}'_1(r) &= a'_1 \boldsymbol{f}_1(r) + a'_2 \boldsymbol{f}_2(r) \qquad & \boldsymbol{f}'_2(r) &= b'_1 \boldsymbol{f}_1(r) + b'_2 \boldsymbol{f}_2(r) \\ \boldsymbol{f}''_1(r) &= a''_1 \boldsymbol{f}_1(r) + a''_2 \boldsymbol{f}_2(r) \qquad & \boldsymbol{f}''_2(r) &= b''_1 \boldsymbol{f}_1(r) + b''_2 \boldsymbol{f}_2(r) \end{aligned} \tag{29}$$

i.e. only wave polarizations depend on r, while the refraction and transmission coefficients are the same at any point of the beam cross section. For nonzero coefficients a and b (polarizations $\boldsymbol{f}'$ and $\boldsymbol{f}''$ include both vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$) the equality $T_1 = T_2 = T_1' = T_2' = T_1'' = T_2''$ should hold true to satisfy expressions (29). If $a_2' = a_2'' = b_1' = b_1'' = 0$, then the waves with different polarizations reflect and refract independently of one another and it should be $T_1 = T_1' = T_1''$, $T_2 = T_2' = T_2''$ to satisfy conditions (29).

One often chooses the beam corresponding to the solutions $T_1 = T_2 = F(r)\hat{1}$ as the incident wave, where F is a scalar function. For example, it can be a Bessel beam, two orthogonal polarizations of which are characterized by the equal radial wavenumbers. Then the reflected and refracted waves have the same solutions $T_1' = T_2' = T_1'' = T_2'' = F(r)\hat{1}$. In the case of the Bessel beams this condition becomes the condition of the radial wavenumber continuity at the interface.

It should be noted that the equality of the integer numbers ν and solutions T for cylindrical beams at the interface between the media 1 and 2 is the analogue of the equality of the phase factors $\exp(\mathrm{i}\psi)$ for the incident, reflected and refracted plane waves. The phases of the plane waves always can be chosen equal, because they are determined by the single wave parameter—the projection of the wavevector onto the interface. Solutions T in different media cannot always be equal to one another, since they include prescribed parameters of the medium. That is why conditions (29) do not fulfil often and one should use more general continuity relationships which contain the superposition of the reflected and transmitted waves. Each partial wave is an exact solution of the Maxwell equations in the appropriate medium. The boundary conditions take the form

$$\begin{pmatrix} \tau(r) \\ \sigma(r) \end{pmatrix} \boldsymbol{a} + \sum_{p=1}^{\infty} \begin{pmatrix} \tau_p'(r) \\ \sigma_p'(r) \end{pmatrix} \boldsymbol{a}_p' = \sum_{p=1}^{\infty} \begin{pmatrix} \tau_p''(r) \\ \sigma_p''(r) \end{pmatrix} \boldsymbol{a}_p''. \tag{30}$$

If the sets of tensor functions τ_p' and τ_p'' form the complete system, then using them we can write an arbitrary function and satisfy the boundary conditions. To determine the constant vectors $\boldsymbol{a}_p'$, $\boldsymbol{a}_p''$ one can decompose r-depending functions into the set of orthogonal functions (exponents or Bessel functions).

Further we will obtain reflection and transmission operators of vector beams. We will derive the operators for the case, when the boundary conditions have the form (27), i.e. conditions (29) hold true. The special cases of such beams are the vector Bessel beams in complex media. Let us consider the incidence of the cylindrical beam from the medium with tensor parameters $\varepsilon^{(0)}$, $\mu^{(0)}$, $\alpha^{(0)}$, $\kappa^{(0)}$ onto the bianisotropic n-layered structure, every layer being characterized by the values $\varepsilon^{(j)}$, $\mu^{(j)}$, $\alpha^{(j)}$, $\kappa^{(j)}$, $j = 1, \ldots, n$. Then the beam falls into the medium with $\varepsilon^{(n+1)}$, $\mu^{(n+1)}$, $\alpha^{(n+1)}$, $\kappa^{(n+1)}$. We solve the boundary problem using evolution operators and impedance tensors introduced above. The transverse field components at the interfaces $z = z_0$ and $z = z_n$ are connected with characteristic matrix of n layers which is equal to the product of evolution operators for each layer:

$$\Omega_{z_0}^{z_n} = \Omega_{z_{n-1}}^{z_n} \Omega_{z_{n-2}}^{z_{n-1}} \cdots \Omega_{z_0}^{z_1}. \tag{31}$$

From the boundary conditions we have

$$\begin{pmatrix} I_z \\ \gamma_{n+1} \end{pmatrix} \boldsymbol{H}_{\perp n+1}(z_n) = \Omega_{z_0}^{z_n} \left[\begin{pmatrix} I_z \\ \gamma_0 \end{pmatrix} \boldsymbol{H}_{\perp 0}(z_0) + \begin{pmatrix} I_z \\ \gamma_0' \end{pmatrix} \boldsymbol{H}'_{\perp 0}(z_0) \right] \tag{32}$$

where γ_0, γ_{n+1}, γ_0' are the surface impedance tensors of the incident $\boldsymbol{H}_{\perp 0}$, transmitted $\boldsymbol{H}_{\perp n+1}$ and reflected $\boldsymbol{H}'_{\perp 0}$ waves, respectively. We introduce the reflection and transmission operators as $\boldsymbol{H}'_{\perp 0} = R\boldsymbol{H}_{\perp 0}$ and $\boldsymbol{H}_{\perp n+1} = D\boldsymbol{H}_{\perp 0}$, then they equal [22]

$$
\begin{aligned}
R &= \left[(-\gamma_{n+1}\ I_z)\Omega_{z_0}^{z_n}\begin{pmatrix} I_z \\ \gamma_0' \end{pmatrix}\right]^- \left[(\gamma_{n+1}\ -I_z)\Omega_{z_0}^{z_n}\begin{pmatrix} I_z \\ \gamma_0 \end{pmatrix}\right] \\
D &= \left[(-\gamma_0'\ I_z)\Omega_{z_n}^{z_0}\begin{pmatrix} I_z \\ \gamma_{n+1} \end{pmatrix}\right]^- (\gamma_0 - \gamma_0').
\end{aligned}
\tag{33}
$$

Thus, the reflection and transmission operators can be calculated using the tensor (matrix) procedures. Reflection and transmission operators allow us to find the amplitudes of the reflected and transmitted fields without dividing them into the eigenwaves.

5. Vector cylindrically symmetric beams in bianisotropic media

For different media we will obtain a number of exact solutions of Maxwell's equations, such as Bessel beams of an integer and fractional order, and cylindrical beams. To determine the transverse field components of the beam in the infinite medium one should substitute the planar tensors η, ζ and vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4$ into expression (18). Incidentally, from (18) it can be easily seen that the fields of all the beams under consideration have an integer topological charge ν, but the solutions may include the Bessel functions of the fractional order.

5.1. Bessel beams of integer order

Let us consider the propagation of the Bessel beam in $\boldsymbol{e}_z$ direction in the homogeneous bianisotropic medium $\varepsilon = \varepsilon_1 I_z + \varepsilon_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z$, $\mu = \mu_1 I_z + \mu_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z$, $\alpha = \kappa = \mathrm{i}\chi \boldsymbol{e}_z^\times$. Tensors $\eta = \eta^{(1)} + \eta^{(2)}$, $\zeta = \zeta^{(1)} + \zeta^{(2)}$ can be presented as follows,

$$
\begin{aligned}
\eta &= J_\nu(q_1 r)\left(\boldsymbol{e}_z - \frac{\mu_2 \nu(\beta - \mathrm{i}k\chi)}{\mu_1 q_1^2 r}\boldsymbol{e}_\varphi\right) \otimes \boldsymbol{e}_z + \frac{\mathrm{i}k\varepsilon_2 J_\nu'(q_2 r)}{q_2}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi \\
\zeta &= -\frac{\mathrm{i}k\mu_2 J_\nu'(q_1 r)}{q_1}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z + J_\nu(q_2 r)\left(\boldsymbol{e}_z - \frac{\varepsilon_1 \nu(\beta + \mathrm{i}k\chi)}{\varepsilon_1 q_2^2 r}\boldsymbol{e}_\varphi\right) \otimes \boldsymbol{e}_\varphi
\end{aligned}
\tag{34}
$$

where $q_1^2 = k^2\varepsilon_1\mu_2 - (k^2\chi^2 + \beta^2)\mu_2/\mu_1$, $q_2^2 = k^2\varepsilon_2\mu_1 - (k^2\chi^2 + \beta^2)\varepsilon_2/\varepsilon_1$. The values q_1 and q_2 are called the transversal (radial) wavenumbers. Usually, one assumes $q_1 = q_2 = q$ and interconnects this parameter with the cone angle for the wavevector of the Bessel beam. The radial wavenumber q is determined by the initial conditions, as the projection of the wavevector of the plane wave onto the interface between two media. For each beam polarization (TE and TM, respectively) the longitudinal wavenumbers equal

$$
\beta_1 = \sqrt{k^2\varepsilon_1\mu_1 - k^2\chi^2 - q^2\mu_1/\mu_2} \qquad \beta_2 = \sqrt{k^2\varepsilon_1\mu_1 - k^2\chi^2 - q^2\varepsilon_1/\varepsilon_2}.
$$

Taking into account the expressions

$$
\boldsymbol{v}_1 = \boldsymbol{v}_4 = 0 \qquad \boldsymbol{v}_2 = \frac{\mathrm{i}k\chi - \beta}{k\mu_1}\boldsymbol{e}_\varphi + \frac{\nu}{k\mu_1 r}\boldsymbol{e}_z \qquad \boldsymbol{v}_3 = \frac{\mathrm{i}k\chi + \beta}{k\varepsilon_1}\boldsymbol{e}_\varphi - \frac{\nu}{k\varepsilon_1 r}\boldsymbol{e}_z
$$

from (18) we find the planar tensors τ and σ describing the transverse electric and magnetic field components of the Bessel beam:

$$
\begin{aligned}
\tau &= \frac{\mu_2(\beta_1 - \mathrm{i}k\chi)}{\mu_1 q}\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b} \otimes \boldsymbol{e}_r + \frac{k\varepsilon_2}{q}\mathrm{e}^{\mathrm{i}\beta_2 z}(\boldsymbol{e}_z \times \boldsymbol{b}) \otimes \boldsymbol{e}_\varphi \\
\sigma &= \frac{k\mu_2}{q}\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b} \otimes \boldsymbol{e}_r + \frac{\varepsilon_2(\beta_2 + \mathrm{i}k\chi)}{\varepsilon_1 q}\mathrm{e}^{\mathrm{i}\beta_2 z}(\boldsymbol{e}_z \times \boldsymbol{b}) \otimes \boldsymbol{e}_\varphi
\end{aligned}
\tag{35}
$$

where

$$\boldsymbol{b} = \mathrm{i}J_\nu'(qr)\boldsymbol{e}_r - \frac{\nu}{qr}J_\nu(qr)\boldsymbol{e}_\varphi. \tag{36}$$

Bessel beam in the considered bianisotropic medium is characterized by two orthogonal polarizations $\boldsymbol{b}$ and $\boldsymbol{e}_z \times \boldsymbol{b}$. These polarizations are quasi-circular and become circular polarizations in the paraxial approximation.

The impedance tensor can be easily calculated using the formula (23); it equals

$$\gamma = \frac{1}{\boldsymbol{b}^2}\left[\frac{k\mu_1}{\beta_1 - \mathrm{i}k\chi}\boldsymbol{b}\otimes\boldsymbol{b} + \frac{\beta_2 + \mathrm{i}k\chi}{k\varepsilon_1}(\boldsymbol{e}_z\times\boldsymbol{b})\otimes(\boldsymbol{e}_z\times\boldsymbol{b})\right]. \tag{37}$$

In the impedance tensor of the backward Bessel beam one need to change the sign of the propagation constant $\beta_{1,2} \to -\beta_{1,2}$. Bianisotropic media possess the property of nonreciprocity, i.e. the forward and backward beams propagate in different ways. This is the distinctive feature of a gyrotropic medium. In anisotropic medium ($\chi = 0$) the impedance tensor of the backward Bessel beam equals $\gamma' = -\gamma$. The case of isotropic medium is realized for $\varepsilon_1 = \varepsilon_2$, $\mu_1 = \mu_2$, $\chi = 0$ and gives well-known relationships.

The evolution operator (25) of the beam in the bianisotropic medium is equal to

$$\begin{aligned}\Omega_0^z = &\begin{pmatrix}\cos(\beta_1 z)I_z & \frac{\beta_1-\mathrm{i}k\chi}{k\mu_1}\sin(\beta_1 z)I_z\\ \frac{k\mu_1}{\beta_1-\mathrm{i}k\chi}\sin(\beta_1 z)I_z & \cos(\beta_1 z)I_z\end{pmatrix}\frac{\boldsymbol{b}\otimes\boldsymbol{b}}{\boldsymbol{b}^2}\\ &+\begin{pmatrix}\cos(\beta_2 z)I_z & \frac{k\varepsilon_1}{\beta_2+\mathrm{i}k\chi}\sin(\beta_2 z)I_z\\ \frac{\beta_2+\mathrm{i}k\chi}{k\varepsilon_1}\sin(\beta_2 z)I_z & \cos(\beta_2 z)I_z\end{pmatrix}\frac{(\boldsymbol{e}_z\times\boldsymbol{b})\otimes(\boldsymbol{e}_z\times\boldsymbol{b})}{\boldsymbol{b}^2}.\end{aligned} \tag{38}$$

The evolution operator is divided into two parts, each of which contains the evolution operator for one of the two orthogonal polarizations. Therefore, we can use 2×2 matrices to describe the evolution of independent waves instead of the block 4×4 matrices. Evolution operator for plane waves has the same form. The single distinction is in vector $\boldsymbol{b}$ which is constant for plane waves and depends on the point in the beam cross section for cylindrically symmetric waves.

The solutions for Bessel beams in considered bianisotropic media are expressed by means of two orthogonal vectors $\boldsymbol{b}$ and $\boldsymbol{e}_z \times \boldsymbol{b}$. Therefore, the conditions (29) for the incident, reflected and refracted waves are satisfied, and we can apply formulae for the reflection and transmission operators (33).

5.2. Fractional Bessel beams

In this subsection, we will consider the typical case of solution of the Maxwell equations in the form of the fractional Bessel beams in an anisotropic medium ($\alpha = \kappa = 0$). Such medium has the following dielectric permittivity and magnetic permeability tensors,

$$\varepsilon = \varepsilon_1\boldsymbol{e}_r\otimes\boldsymbol{e}_r + b\varepsilon_1\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi + \varepsilon_2\boldsymbol{e}_z\otimes\boldsymbol{e}_z \qquad \mu = \mu_1\boldsymbol{e}_r\otimes\boldsymbol{e}_r + b\mu_1\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi + \mu_2\boldsymbol{e}_z\otimes\boldsymbol{e}_z$$

where b is the anisotropy parameter, the same for ε and μ. For each ν there are a couple of eigenwaves with longitudinal wavenumbers

$$\beta_1 = \sqrt{k^2 b\varepsilon_1\mu_1 - \mu_1 q^2/\mu_2} \qquad \beta_2 = \sqrt{k^2 b\varepsilon_1\mu_1 - \varepsilon_1 q^2/\varepsilon_2}$$

and planar tensors

$$\begin{aligned}\tau &= \frac{\mu_2\beta_1}{\mu_1 q}\,\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b}_1\otimes\boldsymbol{e}_r + \frac{k\varepsilon_2}{q}\mathrm{e}^{\mathrm{i}\beta_2 z}(\boldsymbol{e}_z\times\boldsymbol{b}_2)\otimes\boldsymbol{e}_\varphi\\ \sigma &= \frac{k\mu_2}{q}\,\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b}_2\otimes\boldsymbol{e}_r + \frac{\varepsilon_2\beta_2}{\varepsilon_1 q}\mathrm{e}^{\mathrm{i}\beta_2 z}(\boldsymbol{e}_z\times\boldsymbol{b}_1)\otimes\boldsymbol{e}_\varphi\end{aligned} \tag{39}$$

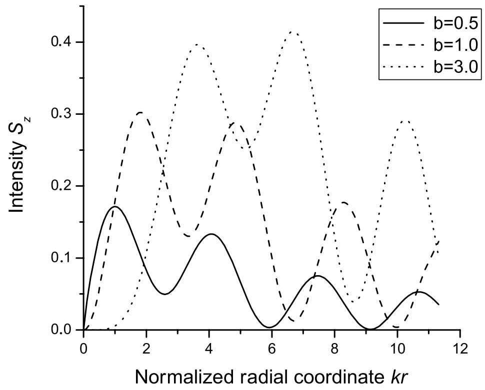

Figure 1. Radial dependence of the Bessel beam intensity for $b = 0.5; 1; 3$. Parameters: $\varepsilon_1 = \varepsilon_2 = 2.0$, $\mu_1 = \mu_2 = 1.2$, $q/k = 1$, $\nu = 2$.

where

$$\boldsymbol{b}_1 = \mathrm{i}J'_{\sqrt{b}\nu}(qr)\boldsymbol{e}_r - \frac{\nu}{qr}J_{\sqrt{b}\nu}(qr)\boldsymbol{e}_\varphi \qquad \boldsymbol{b}_2 = \mathrm{i}J'_{\sqrt{b}\nu}(qr)\boldsymbol{e}_r - \frac{\nu b}{qr}J_{\sqrt{b}\nu}(qr)\boldsymbol{e}_\varphi. \tag{40}$$

These solutions contain Bessel functions of the fractional order for each integer azimuthal number ν. Fractional Bessel beams were obtained experimentally in the papers [5, 7, 8], but those beams have a fractional topological charge ν. In the case under consideration the fractional order of the beam is achieved by means of the medium parameter b. The polarization vectors $\boldsymbol{b}_1$ and $\boldsymbol{e}_z \times \boldsymbol{b}_2$ are not orthogonal and, in general, cannot be written as a linear combination of beam polarizations in an isotropic medium with constant coefficients. That is why we should apply continuity conditions (30) at the interface between the considered anisotropic medium and the isotropic one. So, the introduced reflection and transmission operators (33) can be used for beams in the media with the same anisotropy parameters b or zero azimuthal number ν. Polarization and energy properties of the fractional Bessel beams are investigated below.

In experiments the detector measures the time-averaged energy characteristics. The intensity of the electromagnetic wave is the averaged value of the Poynting vector normal to the detector surface. For the cylindrical beams the intensity is equal to S_z. The averaged value of the Poynting vector takes the following form for each eigenwave (TE and TM, respectively):

$$\begin{aligned} \boldsymbol{S}_1 &= \frac{c}{8\pi}\frac{k\mu_2|c_1|^2}{q}\left(\frac{\mu_2\beta_1}{\mu_1 q}\left(J'^2_{\sqrt{b}\nu} + \frac{\nu^2 b}{q^2r^2}J^2_{\sqrt{b}\nu}\right)\boldsymbol{e}_z + \frac{\nu b}{qr}J^2_{\sqrt{b}\nu}\boldsymbol{e}_\varphi\right) \\ \boldsymbol{S}_2 &= \frac{c}{8\pi}\frac{k\varepsilon_2|c_2|^2}{q}\left(\frac{\varepsilon_2\beta_2}{\varepsilon_1 q}\left(J'^2_{\sqrt{b}\nu} + \frac{\nu^2 b}{q^2r^2}J^2_{\sqrt{b}\nu}\right)\boldsymbol{e}_z + \frac{\nu b}{qr}J^2_{\sqrt{b}\nu}\boldsymbol{e}_\varphi\right). \end{aligned} \tag{41}$$

The energy flux (41) has azimuthal and longitudinal components. The existence of only the azimuthal component in the transverse intensity leads to the closure of the lines of transverse intensity and confirms the diffraction-free nature of the beam. The radial dependence of the beam intensity is shown in figure 1. When the parameter b increases, the intensity maxima shift to the region of large radial coordinates r and the rings become wider. It is the consequence of the increase of the Bessel function order $\sqrt{b}\nu$. For integer values of the order (for example, when b is equal to the square of the integer number) the beam characteristics do not change qualitatively.

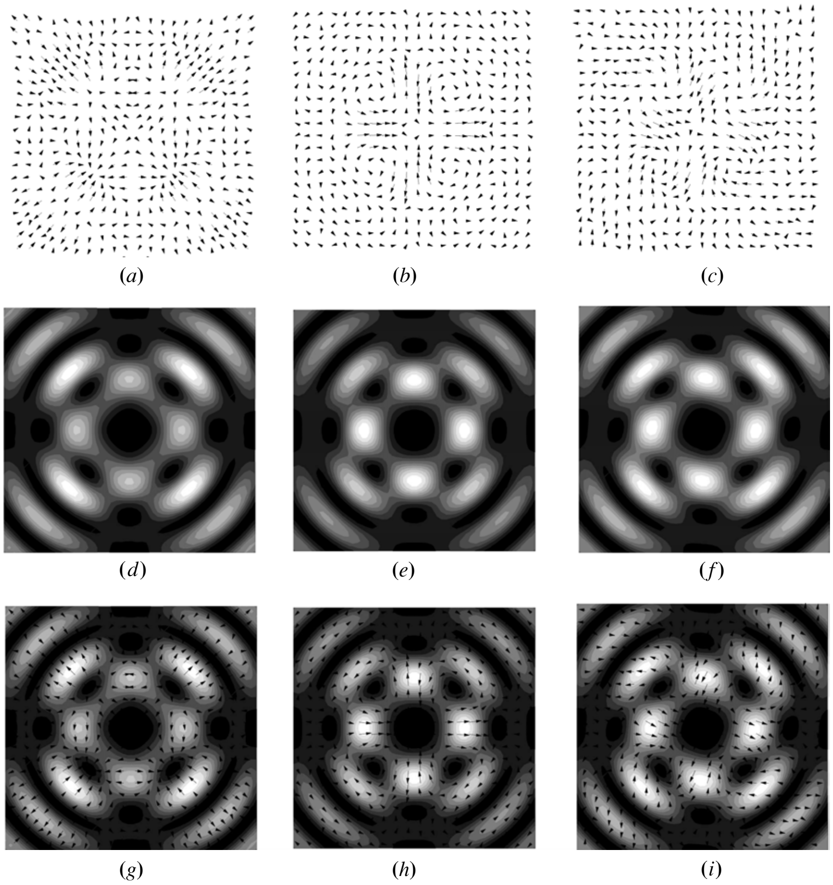

Figure 2. Magnetic field vector distribution ((*a*), (*b*), (*c*)), instantaneous energy flux distribution S_z ((*d*), (*e*), (*f*)) and their overlay ((*g*), (*h*), (*i*)) for Bessel beams with polarizations ((*a*), (*d*), (*g*)) $c_1 \equiv \boldsymbol{a}\boldsymbol{e}_r = 1, c_2 \equiv \boldsymbol{a}\boldsymbol{e}_\varphi = 0$; ((*b*), (*e*), (*h*)) $c_1 = 0, c_2 = 1$; ((*c*), (*f*), (*i*)) $c_1 = 1, c_2 = 1$. Here $\varepsilon_1 = \varepsilon_2 = 2.0, \mu_1 = \mu_2 = 1.2, q/k = 1, \nu = 2, b = 3$, vector $\boldsymbol{a}$ determines the initial field (see formula (18)).

Further, we will consider the magnetic field vector distribution, which is determined by the tensor τ. Such instantaneous field distribution and instantaneous energy flux distribution $S_z = (c/4\pi)[\mathrm{Re}(\boldsymbol{E}) \times \mathrm{Re}(\boldsymbol{H})]\boldsymbol{e}_z$ are given in figure 2. The picture overlay is shown there, too. We see that the structure of the instantaneous intensity has a symmetrical form and is repeated in the angle $\pi/2$ which is determined by the azimuthal number $\nu = 2$. Therefore, for any parameter b and fixed polarization the energy flux distribution is qualitatively the same. One can easily note that for some values of the azimuth φ the magnetic field vectors with different polarizations (a) and (b) become orthogonal, although it is not true in the general case. The fact is that we calculate the real field values and hence we can obtain the orthogonal polarizations, when the scalar product of complex fields becomes imaginary. Exactly this case is realized for $\varphi = \pi m/4$, where m is an integer number. In these directions the most bright energy regions are situated. The centres of the divergent and converging lines of the magnetic field, as well as the centres of the circular lines of force, are situated in the areas of low intensity with azimuths $\varphi = \pi/4 + \pi m/2$. In figure 2(*g*), it is shown that the centres of the divergent and converging lines alternate and magnetic field polarizations in adjacent bright regions are oppositely directed. One can say the same for the beam in figure 2(h). As regards the intensity, the more bright regions are those, in which the magnetic field vectors are directed along the radius. Increasing the anisotropy parameter b we can achieve the increase of the scale of the energy flux distribution. For the eigenwave superposition the polarization vectors rotate creating a more complex spiral structure. The main behaviour of the field lines remains; for instance, the magnetic field vectors of the adjacent bright regions are directed oppositely, too.

5.3. Cylindrical beams

In this subsection, we consider the cylindrically symmetric beams whose solutions cannot be written using only the Bessel functions. For example, such waves appear in the medium ($\alpha = \kappa = 0$)

$$\begin{aligned} \varepsilon &= \varepsilon_1 I_z + \varepsilon_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + b_1\varepsilon_1(\boldsymbol{e}_r \otimes \boldsymbol{e}_z + \boldsymbol{e}_z \otimes \boldsymbol{e}_r) \\ \mu &= \mu_1 I_z + \mu_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + b_1\mu_1(\boldsymbol{e}_r \otimes \boldsymbol{e}_z + \boldsymbol{e}_z \otimes \boldsymbol{e}_r) \end{aligned} \tag{42}$$

where b_1 is a constant parameter. The transverse electric and magnetic field components of the beam are determined by the tensors τ and σ:

$$\begin{aligned} \tau &= \mathrm{e}^{\mathrm{i}\beta_1(z-b_1r)}\frac{\beta_1 a_1}{q}\boldsymbol{d}_1 \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2(z-b_1r)}\frac{k\varepsilon_1 a_2}{q}(\boldsymbol{e}_z \times \boldsymbol{b}) \otimes \boldsymbol{e}_\varphi \\ \sigma &= \mathrm{e}^{\mathrm{i}\beta_1(z-b_1r)}\frac{k\mu_1 a_1}{q}\boldsymbol{b} \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2(z-b_1r)}\frac{\beta_2 a_2}{q}(\boldsymbol{e}_z \times \boldsymbol{d}_2) \otimes \boldsymbol{e}_\varphi \end{aligned} \tag{43}$$

where $\boldsymbol{b}$ is the vector (36),

$$\begin{aligned} \boldsymbol{d}_1 &= \boldsymbol{b} - \frac{b_1 q}{\beta_1 a_1}J_\nu(qr)\boldsymbol{e}_r \qquad & \boldsymbol{d}_2 &= \boldsymbol{b} - \frac{b_1 q}{\beta_2 a_2}J_\nu(qr)\boldsymbol{e}_r \\ a_1 &= \mu_2/\mu_1 - b_1^2 & a_2 &= \varepsilon_2/\varepsilon_1 - b_1^2. \end{aligned}$$

The longitudinal wavenumbers of TE and TM waves have the form

$$\beta_1 = \sqrt{k^2\varepsilon_1\mu_1 - q^2/a_1} \qquad \beta_2 = \sqrt{k^2\varepsilon_1\mu_1 - q^2/a_2}. \tag{44}$$

The exponent indicates the variation of the beam phase not only in longitudinal and azimuthal directions, but also in the radial. At the interface with an isotropic medium one should use the field continuity conditions (30), because one of the solutions contains the factor $\exp(-\mathrm{i}b_1\beta r)$, while another solution does not. The eigenwave polarization vectors are not orthogonal, which is caused by the anisotropy b_1.

The energy flux for both TE and TM waves has two components (longitudinal and azimuthal) and takes the form (41), if in this formula μ_2 and ε_2 are replaced by $\mu_2 - b_1^2\mu_1$ and $\varepsilon_2 - b_1^2\varepsilon_1$, respectively. The direction of the azimuthal energy flux can be varied into the opposite one, if the signs of the medium parameters a_1 and a_2 are changed. The same result can be obtained for an evanescent wave. The evanescent wave is characterized by the imaginary propagation constant $\beta_{1,2} = \mathrm{i}\beta'_{1,2}$, the energy flux for each eigenwave taking the form

$$\begin{aligned} \langle \boldsymbol{S}_1 \rangle &= \frac{c}{8\pi}\mathrm{e}^{-2\beta'_1(z-b_1r)}\frac{k\nu\mu_1 a_1|c_1|^2}{q^2 r}J_\nu^2(qr)\boldsymbol{e}_\varphi \\ \langle \boldsymbol{S}_2 \rangle &= \frac{c}{8\pi}\mathrm{e}^{-2\beta'_2(z-b_1r)}\frac{k\nu\varepsilon_1 a_2|c_2|^2}{q^2 r}J_\nu^2(qr)\boldsymbol{e}_\varphi. \end{aligned} \tag{45}$$

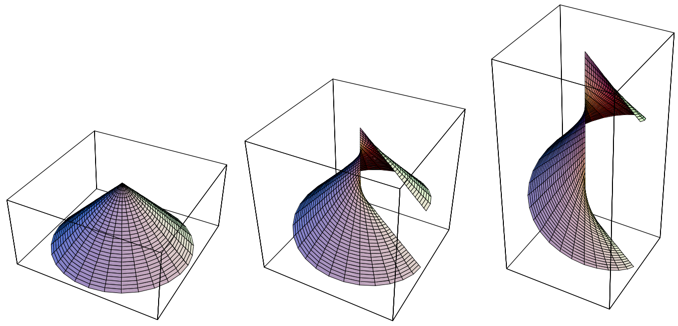

Figure 3. Wave front of the cylindrical beam in medium (42) for parameter $b_1 = -3$ and topological charge (*a*) $\nu = 0$, (*b*) $\nu = 1$, (*c*) $\nu = 3$.

The summand $2b_1\beta' r$ in the exponent argument is of great importance. During the wave propagation along the z axis the wave amplitude exponentially vanishes. At the same time, the radial exponential factor may both decrease (for $b_1 < 0$) and increase (for $b_1 > 0$) the beam energy flux. The second case gives infinite fields for $r = \infty$ and cannot be realized. On the other hand, there are no beams of infinite radius. For a beam of finite size (for example, in the fibre) such amplitude rise for large r is possible. From (44) it follows that for $a_{1,2} < 0$ the longitudinal wavenumber is always the real value and there are no evanescent waves for any radial wavenumber q. The evanescent waves appear only for $|b_1| < \mu_2/\mu_1$ and $|b_1| < \varepsilon_2/\varepsilon_1$.

Further, we will obtain the electric and magnetic field strengths in the paraxial approximation. For the sake of simplicity, we assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $\mu_1 = \mu_2 = \mu$ (therefore, $\beta_1 = \beta_2 = \beta$). In the paraxial approximation the electromagnetic field of the cylindrical beam becomes transversal and can be determined from (43) at $q \to 0$:

$$\begin{aligned} \boldsymbol{H} &= \exp(\mathrm{i}\beta z - \mathrm{i}b_1\beta r + \mathrm{i}(\nu-1)\varphi)\left(1-b_1^2\right)(\mathrm{i}\beta c_1 + k\varepsilon c_2)J_{\nu-1}(qr)\boldsymbol{e}_+ \\ \boldsymbol{E} &= \exp(\mathrm{i}\beta z - \mathrm{i}b_1\beta r + \mathrm{i}(\nu-1)\varphi)\left(1-b_1^2\right)(\mathrm{i}\beta c_2 - k\mu c_1)J_{\nu-1}(qr)\boldsymbol{e}_+ \end{aligned} \tag{46}$$

where $\boldsymbol{e}_+ = (\boldsymbol{e}_x + \mathrm{i}\boldsymbol{e}_y)/\sqrt{2}$. We see that the beam polarization in the paraxial approximation is the circular one. In essence, such a solution describes a complex scalar wave. The nth order beam corresponds to the solution for $\nu = n + 1$. Hence, the scalar wave can be written in the form $\psi = \exp(\mathrm{i}\beta z - \mathrm{i}b_1\beta r + \mathrm{i}n\varphi)J_n(qr)A$. For $n \neq 0$ such beam has a singularity at the point $r = 0$, where the wavefunction becomes zero. Therefore, an optical vortex [23–25] appears near this point. The vortex topological charge (dislocation strength) is defined by circulation of the phase gradient around the singularity and is equal to the integer number n. The topological charge determines the vortex orbital angular momentum, which is caused by the screw wavefront structure. The main difference of the considered scalar wave ψ from the conventional paraxial Bessel beam is connected with the phase $-\mathrm{i}b_1\beta r$. Such value leads to the unusual equiphase surface. For $\nu = 0$ the wavefront presents the divergent (converging) cone. The azimuthal component of the phase results in the cone twisting. The cone itself breaks and becomes the expanding helix when topological charge increases (see figure 3). In a similar manner one can trace the change of the equiphase surface for different anisotropy parameters b_1. For $b_1 = 0$ and $\nu \neq 0$ the wavefront is the helical surface, as for a scalar Bessel beam. Nonzero anisotropy parameter leads to the untwisting of the helix. The energy flux of the scalar wave (Poynting vector) is proportional to the phase gradient and equals $\boldsymbol{S} = J_n^2|A|^2(\beta \boldsymbol{e}_z - b_1\beta \boldsymbol{e}_r + n\boldsymbol{e}_\varphi/r)$. It is directed normally to the phase wavefront. The helical structure of the phase front is caused by the azimuthal energy flux, while the cone-shaped phase front is determined by the radial component of **S**.

6. Conclusion

In summary, we have presented a tensor description of the vector cylindrically symmetric beams (including Bessel beams) which correspond to exact solutions of Maxwell's equations in bianisotropic media. Reflection and transmission operators used earlier for plane waves are proved to be applicable to the cylindrical beams. We have found that to satisfy the boundary conditions in general one needs to write the superposition of reflected and refracted waves, each partial wave being the solution of the wave equation in the appropriate medium. We have obtained the fractional Bessel beams with integer topological charge as solutions of the Maxwell equations in anisotropic media. The beam fractional order is caused by the transverse anisotropy of the medium. In this paper, we have considered cylindrical beams with the radial dependence in the form of the product of the Bessel function and the exponent. For such beams the anisotropy can vary the direction of the azimuthal energy flux and determines the equiphase surface in the form of the divergent (converging) helix.

We believe that the results obtained can be applied for creating cylindrically symmetric beams with prescribed characteristics of the intensity and polarization distributions. Theoretical solutions (46) can prove to be useful for optically manipulating microparticles and atoms. The energy flux of the beams has the radial component which gathers particles at centre $r = 0$ or shifts them out of centre depending on sign of the parameter b_1.

Generally speaking, there are bianisotropic media, cylindrical waves in which are expressed using the Laguerre polynomials. Such beams propagate in anisotropic media, for which both matrices P and Q in equation (10) have the diagonal form. However, this will be the subject of further investigation.

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